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NUMERICAL EVALUATION OF GEOMETRICAL OPTICS
RADAR CROSS SECTION

MAY 1968

M. R. Weiss

Work Performed for

ADVANCED RESEARCH PROJECTS AGENCY

Contract Administered by

DEVELOPMENT ENGINEERING DIVISION
DIRECTORATE OF PLANNING AND TECHNOLOGY

ELECTRONIC SYSTEMS DIVISION

AIR FORCE SYSTEMS COMMAND

UNITED STATES AIR FORCE

L. G. Hanscom Field, Bedford, Massachusetts



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Project 8051

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts

Contract AF19(628)-5165

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FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for Advanced Research Projects Agency; the contract was monitored by the Directorate of Planning and Technology, Electronic Systems Division, of the Air Force Systems Command under Contract AF 19(628)-5165.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

A. P. TRUNFIO
Project Officer
Development Engineering Division
Directorate of Planning & Technology

ABSTRACT

A numerical technique for evaluation of the geometrical optics radar cross section of a general doubly curved convex conducting body is described. A surface fitting method is employed where unique surfaces are established utilizing the Schmidt orthogonalization technique for matrix inversion. In this manner, linearly dependent data points are discarded prior to their inclusion in the surface fitting procedure.

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SECTION I

INTRODUCTION

A technique is presented for the numerical evaluation of the geometrical optics mono-static radar cross section of a general doubly curved convex body. This computer program requires as inputs the cartesian coordinates of a number of points of the given surface. Provided as outputs are the scattered amplitude for prescribed incident directions (θ, φ) .

SECTION II

DISCUSSION

ANALYTIC PROCEDURE

The calculation of the geometrical optics radar cross section of a general doubly curved convex body requires a calculation of the gaussian curvature as a function of normal direction of the given surface,

$$\sigma(\theta, \phi) = \pi K^{-1}(\theta, \phi) \quad (1)$$

where $K(\theta, \phi)$ is the gaussian curvature where the surface normal is at angles θ and ϕ . Most direct approaches to the calculation of gaussian curvature entail an evaluation of first and second derivatives at various points on the surface with a corresponding finite difference approach in the case of numerical approximation. In order to achieve acceptable accuracies with finite differences, however, a large number of closely spaced points describing the surface are necessary. This difficulty can be circumvented by a surface fitting procedure which allows the gaussian curvature to be calculated directly from the fitted surface.

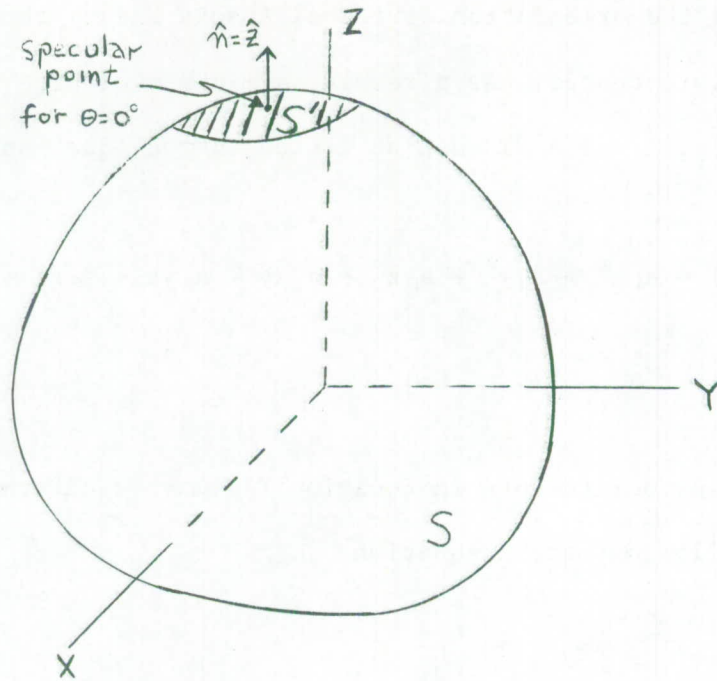


Figure 1. General Convex Body

Consider the general convex surface, denoted by S , shown in Figure 1. For a viewing direction of $\theta = 0^\circ$ the surface, S' , in the neighborhood of the specular point may be approximated by a quadric surface (for this viewing direction the specular point is that point with the largest z value). In practice, S' can be approximated by fitting the set of cartesian points in the vicinity of the specular point to a general quadric surface.

To describe S' by a quadric, nine coefficients are required; these coefficients in the case of a general ellipsoid are related to the 1) three semi-axes of the ellipsoid, 2) the Euler angles

describing the orientation of the ellipsoid and 3) the coordinate of the ellipsoid center. As a result, a minimum of nine surface points $(x_i, y_i, z_i, i = 1 - 9)$ must be fitted to the equation:

$$g(x,y,z) = a_1 x^2 + a_2 y^2 + a_3 z^2 + a_4 xy + a_5 yz + a_6 xz + a_7 x + a_8 y + a_9 z - 1 \equiv 0 \quad (2)$$

The unknown coefficients in Equation (1) may be determined by solution of the following matrix equation:

$$RA = I \quad (3)$$

where I is a column matrix with each entry unity, and

$$A = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_9 \end{bmatrix}$$

$$R = \begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1 y_1 & y_1 z_1 & x_1 z_1 & x_1 & y_1 & z_1 \\ x_2^2 & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ x_9^2 & . & . & . & . & . & . & . & . \end{bmatrix}$$

Equation (3) may be solved for A provided that R^{-1} is non-singular. The cases where the inverse of R is singular indicate that no unique surface exists for the given set of nine points. In these cases the multiple solutions involve degenerate surfaces (planes, intersection of planes, etc.,) in addition to the actual quadric surface. One such example might be nine points lying on a spheroid but whose z values are identical; obviously the plane $a_9 z - 1 = 0$ satisfies the nine points equally well. Since it is extremely difficult to predict the presence of multiplicative solutions on the basis of the x,y,z points themselves, the following orthogonalization technique^[1] has been employed for the solution of Equation (2).

If we consider the individual rows of R as being nine-component vectors,

$$\vec{R}_i = \hat{e}_1 x_i^2 + \hat{e}_2 y_i^2 + \hat{e}_3 z_i^2 + \hat{e}_4 x_i y_i + \hat{e}_5 y_i z_i + \hat{e}_6 x_i z_i + \hat{e}_7 x_i + \hat{e}_8 y_i + \hat{e}_9 z_i$$

$$i = 1, 2 \dots 9$$

we may form a new matrix, by linear transformation of R, such that the new row "vectors" are orthonormal. This is most easily done by the Schmidt orthogonalization technique where in terms of \vec{R}_i the new vectors are given by:

$$\vec{N}_1 = \vec{R}_1 / |\vec{R}_1| \quad (4)$$

$$\vec{N}_i = \frac{\vec{R}_i + B_{i,(i-1)}\vec{N}_{i-1} + B_{i,(i-2)}\vec{N}_{i-2} + \dots + B_{i,1}\vec{N}_1}{|\vec{R}_i + B_{i,(i-1)}\vec{N}_{i-1} + B_{i,(i-2)}\vec{N}_{i-2} + \dots + B_{i,1}\vec{N}_1|} \quad (4)$$

where

$$B_{i,j} = - \vec{R}_i \cdot \vec{N}_j .$$

We may now write Equation (3) in terms of the new matrix N,

$$NA = C \quad (5)$$

where C is formed by the same linear transformations as N, i.e.,

$$C_i = \frac{1 + B_{i,(i-1)}C_{i-1} + B_{i,(i-2)}C_{i-2} + \dots + B_{i,1}C_1}{|\vec{R}_i + B_{i,(i-1)}\vec{N}_{i-1} + B_{i,(i-2)}\vec{N}_{i-2} + \dots + B_{i,1}\vec{N}_1|} \quad (6)$$

Since the inverse of N is simply given as its transpose we have,

$$A = N' C . \quad (7)$$

The advantage of solving the original set of simultaneous equations in this manner is that dependent equations are exposed before their inclusion in the solution of A. In the construction of N any point (x_i, y_i, z_i) which will produce a non-unique surface (i.e. result in a dependent equation) will yield a null vector for \vec{N}_i .

As a result of this indication, linearly dependent equations can be discarded and additional surface points introduced until nine linearly independent equations result.

For a general convex surface, $g(x,y,z) = 0$, the gaussian curvature at the point $\hat{n} = \hat{z}$ ($\theta = 0^\circ$) is easily shown to be^[2]

$$K = \frac{\left(\frac{\partial^2 g}{\partial x^2}\right) \left(\frac{\partial^2 g}{\partial y^2}\right) - \left(\frac{\partial^2 g}{\partial x \partial y}\right)^2}{|\text{grad}g|^2} \quad (8)$$

where all derivatives are evaluated at the specular point. For the fitted quadric given by Equation (2) we have:

$$K = \frac{4a_1 a_2 - a_4^2}{[2a_3 z + a_5 y + a_6 x + a_9]^2} \bigg|_{\text{specular point}} \quad (9)$$

What remains to be determined is the exact location of the specular point x_s , y_s , and z_s . Since we are interested in the specular point for $\theta = 0$, the following simultaneous equations may be used:

$$\begin{aligned} g(x,y,z) &= 0 \\ \frac{\partial g}{\partial x} &= 2a_1 x + a_4 y + a_6 z + a_7 \equiv 0 \\ \frac{\partial g}{\partial y} &= 2a_2 y + a_4 x + a_5 z + a_8 \equiv 0 \end{aligned} \quad (10)$$

NUMERICAL PROCEDURE

The analytic results previously described have been incorporated in a computer program designed to provide the geometrical optics radar cross section for an arbitrary convex body at prescribed incident angles. A flow diagram indicating the necessary subroutines and their purpose is shown in Figure 2. A description of the required steps is given as follows:

a) A series of x, y, z points, referenced to a fixed origin located within the body, describing the shape are necessary as inputs. These points may be coarsely spaced when the curvature of the body is small, the density of points increasing for larger curvature regions. Points describing the body may be randomly chosen on the surface; it is only necessary that the series of points provide a reasonable representation of the body.

b) The initial step allows the body to be rotated to any prescribed viewing direction. This is accomplished by transforming the x, y, z coordinates of the input points such that the new z -direction coincides with the given viewing direction. It is assumed that the original points are given such that the z -axis refers to $\theta = 0^\circ$, and $\phi = 0^\circ$ is the equation of the x - z half plane, hence for this incident direction no transformation is required. As a matter of procedure the computer program fixes the ϕ orientation and performs the radar cross section calculations to follow for each of the prescribed θ values. Consequently, for each of these calculations

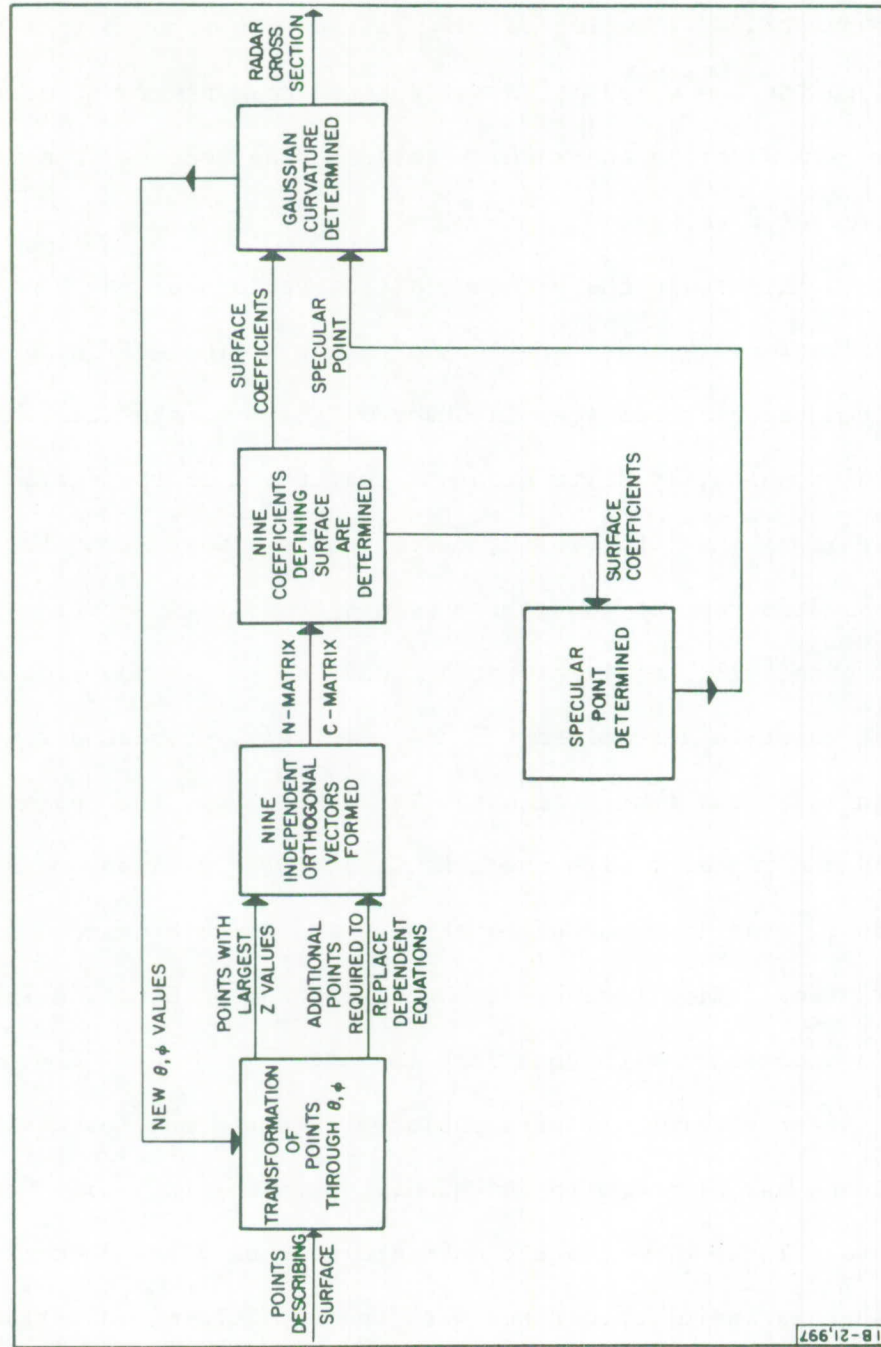


Figure 2. Flow Chart of Necessary Subroutines

the input points are rotated by the appropriate θ value (only the x and z values of the input points are altered for these transformations). Having completed this series of calculations the input points are returned to the $\theta = 0^\circ$ position and rotated to a new ϕ orientation (in this case altering the x and y values), the process then continuing for the set of θ values.

c) At this stage the orthonormal row vectors of the N matrix are formed. These vectors are formed one at a time utilizing surface points from the previous step in order of their z magnitude. Since the specular point, by definition, is that point on the surface with largest z magnitude (the viewing direction is always along the z-axis) this process guarantees that those points nearest in location to the specular point will be fitted to the quadric. If at any time a dependent point is determined (such a point will produce a vector \vec{N}_i , with magnitude less than some established minimum) this point is discarded and replaced with the next largest (in z-value) point. As a result, nine independent equations, and hence a unique quadric are guaranteed. Simultaneously, the right hand of Equation (4) is formed in accordance with Equations (3) and (5).

d) At this point, by straightforward matrix multiplication, the coefficients associated with the quadric surface $g(x,y,z) = 0$ are determined. These nine coefficients are then used for determination of the specular point associated with the now determined surface (for $\theta = 0^\circ$) as follows:

e) The x, y, z coordinates of the specular point are determined by solution of Equation (10). These three equations, two linear and one non-linear, can be solved by using the two linear equations for determining x and y in terms of z and then solving the resultant quadratic equation for z .

f) Equation (9) may now be used for determining the gaussian curvature [and hence the desired radar cross section from equation (1)] for the aspect angles determined from step (a).

g) At this juncture a new set of aspect angles can be specified resulting in the transformation described in (a) with steps (b) through (f) being repeated in identical fashion. In this manner the radar cross section can be conveniently determined for the full set of viewing directions in θ and ϕ .

SECTION III

NUMERICAL RESULTS

For the purposes of demonstration, radar cross section calculations have been computed for a 2:1 prolate spheroid. Since this shape is a special case of an ellipsoid, the surface fitting, in principal, can be handled exactly by the method described. The results, nevertheless, do indicate the accuracy lost in the matrix inversion and specular point calculations and the efficiency of the program in general.

Thirty-seven points using 6 digit accuracy were used to describe the spheroid; these points are listed in Table I.

The resulting radar cross section is shown in Table II. These results are listed together with results of an exact radar cross section computation^[3] and one using exact geometrical optics.

A comparison of columns one and two in Table II indicates that 1-2 digits are lost in the process of curve fitting. Higher accuracy, however, could be achieved either by using more digits for the input points or by using a higher density of points to describe the spheroid. The nature of the geometrical optics approximation, however, does not warrant a higher accuracy than that shown in Table II.

Table I
2:1 Prolate Spheroid

x	y	z
.000000	.000000	2.000000
.100000	.100000	1.979900
-.100000	.100000	1.979900
.000000	-.297909	1.909190
-.210654	-.210654	1.909190
-.297909	.000000	1.909190
.210654	-.210654	1.909190
-.250000	-.250000	1.870830
.250000	.250000	1.870830
.000000	-.500000	1.732050
.000000	.500000	1.732050
-.500000	-.500000	1.414210
-.500000	.500000	1.414210
.500000	.500000	1.414210
.500000	-.500000	1.414210
.707107	.000000	1.414210
.000000	.707107	1.414210
.000000	-.707107	1.414210
1.000000	.000000	.000000
.000000	-.707107	-1.414210
.000000	.707107	-1.414210
.707107	.000000	-1.414210
.500000	-.500000	-1.414210
.500000	.500000	-1.414210
-.500000	.500000	-1.414210
-.500000	-.500000	-1.414210
.000000	.500000	-1.732050
.000000	-.500000	-1.732050
.250000	.250000	-1.870830
-.250000	-.250000	-1.870830
.210654	-.210654	-1.909190
-.297909	.000000	-1.909190
-.210654	-.210654	-1.909190
.000000	-.297909	-1.909190
-.100000	.100000	-1.979900
.100000	.100000	-1.979900
.000000	.000000	-2.000000

Table II
Radar Cross Section Results

$$\sigma / \pi b^2$$

Aspect Angle	Computed Geometrical Optics	Exact Geometrical Optics	Exact*	
			Horizontal	Vertical
0°	.250022	.250000	.2514	.2514
10	.261715	.261706	.3018	.3272
20	.300357	.300399	.3730	.2932
30	.378677	.378698	.3286	.4204
40	.524883	.524925	.5576	.5388
50	.797513	.797537	.7768	.8016
60	1.30620	1.30612	1.360	1.313
70	2.19221	2.19180	2.173	2.140
80	3.36393	3.36394	3.424	3.280
90	4.00001	4.00000	4.080	3.860

*Calculated at $kb = 10$

The simplicity of the geometrical optics computation has allowed the data in column one to be generated in about 10 seconds of computer time. In addition, the geometrical optics approximation applies in a region of body size where exact calculations tend to become impractical requiring long running times.

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